DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

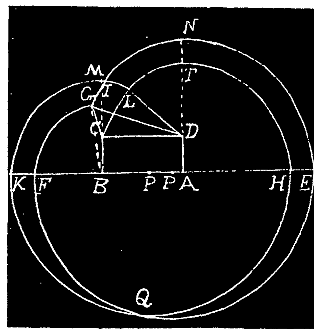
ARITHMETIC.

107. Proposed by REBER V. ALLEN, Hooker Station, Ohio.

A barn, $ABCD$, length $AB=b$ feet, width $AD=a$ feet, standing in an open field, has a horse tethered to a point, P, in the side, AB, distance $AP=c$ feet, with a rope R feet long. Over what area can the horse graze ?

I. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

We might consider six cases of which only the fifth and sixth are represented in the figure.



Case I. In every case $c<b-c$. $R>c$ and $<b-c$, also $<a+c$.

Area$=\frac{1}{2}\pi R^2+\frac{1}{4}\pi(R-c)^2=\frac{1}{4}\pi(3R^2-2Rc+c^2)$.

Case II. $R>c$, $R>b-c$, $R<a+c$, $R<a+b-c$.

Area$=\frac{1}{2}\pi R^2+\frac{1}{4}\pi(R-c)^2+\frac{1}{4}\pi(R-b+c)^2=\frac{1}{4}\pi(4R^2+2c^2+b^2-2Rb-2bc)$.

Case III. $R>c+a$, $R<a+b-c$.

Area$=\frac{1}{2}\pi R^2+\frac{1}{4}\pi(R-c)^2+\frac{1}{4}\pi(R-c-a)^2+\frac{1}{4}\pi(R-b+c)^2=\frac{1}{4}\pi(5R^2-2Rc+3c^2+a^2-2Ra+2ac+b^2-2Rb-2bc)$.

Case IV. $R>c+a$, $R>a+b-c$, $R<a+b$.

Area$=\frac{1}{2}\pi R^2+\frac{1}{4}\pi(R-c)^2+\frac{1}{4}\pi(R-c-a)^2+\frac{1}{4}\pi(R-b+c)^2+\frac{1}{4}\pi(R-a-b+c)^2=\frac{1}{2}\pi(3R^2+2c^2+a^2-Ra+b^2-2Rb-2bc+ab)$.

Case V. $R>a+b$; intersection at L between AD, BC produced.

Area$=\frac{1}{2}\pi R^2+\frac{1}{4}\pi(R-c)^2+\frac{1}{4}\pi(R-b+c)^2+$sector $DTL+$sector $CIL+$triangle DLC.

$DL=R-a-c$, $DC=b$, $CL=(R-a-b+c)$.

$$\therefore\ \angle LDC=\theta=\cos^{-1}\left(\frac{b^2+(R-a-c)^2-(R-a-b+c)^2}{2b(R-a-c)}\right)$$

$$\angle DCL=\varphi=\cos^{-1}\left(\frac{b^2+(R-a-b+c)^2-(R-a-c)^2}{2b(R-a-b+c)}\right).$$

$\therefore$ Area$=\frac{1}{4}\pi(4R^2+2c^2+b^2-2Rb-2bc)+\frac{1}{2}(R-a-c)^2(\frac{1}{2}\pi-\theta)+\frac{1}{2}(R-a-b+c)^2(\frac{1}{2}\pi-\varphi)+\frac{1}{2}b(R-a-c)\sin\theta$.

Case VI. $R>a+b$; intersection in angle KBM.

Area$=\frac{1}{2}\pi R^2+\frac{1}{4}\pi(R-c)^2+$sector $NDG+$sector $FBG+$triangle $CDG+$triangle CBG.

$DG=(R-a-c)$, $DC=b$, $BC=a$, $BG=(R-b+c)$, $DB=\sqrt{(a^2+b^2)}$.

$$\therefore \angle DBG=\beta=\cos^{-1}\left(\frac{a^2+b^2+(R-b+c)^2-(R-a-c)^2}{2(R-b+c)\sqrt{(a^2+b^2)}}\right)$$

$$\angle GDB=\gamma=\cos^{-1}\left(\frac{a^2+b^2+(R-a-c)^2-(R-b+c)^2}{2\sqrt{(a^2+b^2)}(R-a-c)}\right)$$

$$\angle CBD=\beta'=\cos^{-1}\left(\frac{a}{\sqrt{(a^2+b^2)}}\right)=\tan^{-1}\left(\frac{b}{a}\right)$$

$\angle CDB=\frac{1}{2}\pi-\beta'$, $\angle GDC=\gamma+\beta'-\frac{1}{2}\pi$, $\angle CBG=\beta-\beta'$, triangle DCG $+$triangle $BCG=$triangle $DGB-$triangle DCB.

$\therefore$ Triangle $DCG+$triangle $BCG=\frac{1}{2}\sqrt{(a^2+b^2)}(R-a-c)\sin\gamma-\frac{1}{2}ab$.

$\therefore$ Area$=\frac{1}{4}\pi(3R^2-2Rc+c^2)+\frac{1}{2}(R-a-c)^2(\pi-\gamma-\beta')+\frac{1}{2}(R-b+c)^2(\beta-\beta')+\frac{1}{2}\sqrt{(a^2+b^2)}(R-a-c)\sin\gamma-\frac{1}{2}ab$.

II. Solution by C. C. BEBOUT, Professor of Mathematics in Elgin High School, Elgin, Ill.; CHARLES C. CROSS, Libertytown, Md., and ELMER SCHUYLER, High Bridge, N. J.

As shown by the figure, the area grazed over is made up of one semi-circle, two quadrants, two sectors, and a triangle. We must sum the areas of these.

Area of semi-circle $HPK=\frac{1}{2}\pi R^2$ ......(1).

Area of quadrant $HAK=\frac{1}{4}\pi(R-c)^2$ ......(2).

Area of quadrant $KBI=\frac{1}{4}\pi(R-b+c)^2$ ..... (3).

By using formula $A=\sqrt{[s(s-a)(s-b)(s-c)]}$ we get area of triangle DLC $=\sqrt{[c(R-a)(R-a-b)(b-c)]}$ ......(4).

To find the areas of the sectors we must find the angles TDL and ICL. These angles are respectively the complements of angles LDC and DCL, which may be found as angles of the triangle LCD, whose sides are known. To find these angles use the trigonometric formula:

$$\tan\tfrac{1}{2}A=+\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}.$$

Then $\tan\frac{1}{2}\angle LDC=\sqrt{\dfrac{c(R-a-b)}{(R-a)(b-c)}}$ and $\tan\frac{1}{2}\angle LCD=\sqrt{\dfrac{(R-a-b)(b-c)}{c(R-a)}}$

$$\therefore \angle LDT=(90^\circ-\angle LDC)=90^\circ-2\tan^{-1}\sqrt{\frac{c(R-a-b)}{(R-a)(b-c)}},$$

$$\text{and } \angle LCI=(90^\circ-\angle LCD)=90^\circ-2\tan^{-1}\sqrt{\frac{(R-a-b)(b-c)}{c(R-a)}}.$$

$$\therefore \text{ Area of sector } LDT=\frac{360^\circ}{90^\circ-2\tan^{-1}\sqrt{\dfrac{c(R-a-b)}{(R-a)\,(b-c)}}}[\pi(R-a-c)^2]\ldots\ldots(5),$$

$$\text{and area of sector } LCI=\frac{360^\circ}{90^\circ-2\tan^{-1}\sqrt{\dfrac{(R-a-b)(b-c)}{c(R-a)}}}[\pi(R-a-b+c)^2]\,..(6).$$

Summing (1), (2), (3), (4), (5), and (6), we get the area grazed over,

$$\tfrac{1}{4}\pi(4R^2+b^2+2c^2-2bR-2bc)+\sqrt{[c(R-a)(R-a-b)(b-c)]}$$

$$+\frac{360^\circ}{90^\circ-\tan^{-1}\sqrt{\dfrac{c(R-a-b)}{(R-a)\,(b-c)}}}[\pi(R-a-c)^2]$$

$$+\frac{360^\circ}{90^\circ-2\tan^{-1}\sqrt{\dfrac{(R-a-b)(b-c)}{c(R-a)}}}[\pi(R-a-b+c)^2].$$

Solutions of problem 106 were received from P. S. Berg and Elmer Schuyler, and of problem 105 from Sylvester Robins. These solutions came too late for credit in last issue.

NOTE ON THE CALCULATION OF INTEREST AND DISCOUNT.

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In the January number of the MONTHLY, Hon. J. H. Drummond, after giving the answer to the problem, 'What are the proceeds of a note discounted at a bank for 10 years at 10 per cent.,' as *nothing*, says, "The method of calculating discount used by banks was invented to evade the usury laws. I have thought that the court which first sustained the method could not have been well versed in mathematical principles."

A curious thing and commentary on the preceding is that all methods of calculating interest and discount are open to objection. Custom requires that interest be paid at the end of each year or specified fraction of a year. When it is not paid until the end of a period of two or more years, the lender is defrauded of the interest on his interest. If all interest payments are made promptly, it is equivalent to paying compound interest. The business world recognizes that compound interest is the only fair kind. Thus tables of bond values and the like are always made on a compound interest basis. But simple interest is